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Holographic description of heavy-quark potentials in an inflationary braneworld scenario

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ABSTRACT: We study heavy-quark potential in an inflationary braneworld scenario. The scenario we consider is an (Euclidean) AdS_4 inflating braneworld embedded in a static Euclidean AdS_5 . Using gauge/gravity duality we obtain a confining phase depending on the ratio between the Hubble parameter H in the braneworld and the brane tension σ .

KEYWORDS: Confinement, Gauge-gravity correspondence, QCD

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1 Introduction

The holographic computation of heavy quark potentials [1–8] is formulated by relating the string world-sheet area along the bulk with the area of the Wilson loop whose contour lies on the p -dimensional boundary of the bulk. The bulk is described by the geometry of a stack of a large number N of Dp -branes in ten-dimensional spacetime. Following the usual Euclidean lattice QCD¹ formulation for the confining regime, one usually assumes the Euclidean signature to evaluate both the Wilson loop and the string world-sheet area [1, 5, 11–13], although similar evaluations in Lorentzian signature are also possible [14]. For AdS/CFT correspondence [1] one finds a Coulomb-like potential. However, in order to find a gravity dual for a confining theory, such as QCD, one should ‘deform’ the AdS geometry of the bulk [6, 7, 10, 15–17]. The deformation we assume here is a deviation from the conformal case that is *naturally* achieved in geometries of brane cosmology scenarios [18, 19]. In order to keep our setup in the string theory framework, this deformation is such that almost all of the well-known AdS/CFT correspondence is maintained. As we shall see later, this is a slight deviation around AdS space controlled by the tension and the cosmological constant on the brane. We have found that such a deviation implies a *confining regime* in the dual gauge field theory that is directly associated with the cosmological

¹For a recent and comprehensive review on QCD from the AdS/CFT point of view see [9]. For an interesting criticism on this point of view see [10].

constant on the brane. Indeed, this is a general class of geometries where the brane has a cosmological constant, i.e., a *bent* brane, such as AdS_4 or dS_4 branes [20–26], in contrast with flat branes in the original AdS/CFT correspondence.

In this paper we shall assume an inflationary phase on the brane [18, 19, 24, 27–30]. In order to compute a Wilson loop on an inflationary brane we euclideanize the time — for further details on Euclidean brane cosmology see [31, 32]. For consistency, the inflationary scenario is equivalent to assume an ‘Euclidean AdS_4 ’ brane with negative cosmological constant $\bar{\Lambda}$. We shall use the well-known brane cosmology metric [18, 19]. In this case the Nambu-Goto action and the interquark distance scale differently with time. We show that strings live in a static background in the bulk, whereas their endpoints describe a gauge field theory on the boundary with an Euclidean AdS_4 metric. Thus, in order to compute the static quark potential on the boundary, the Wilson loop is evaluated in the “comoving frame”.

The main result we obtain in this work is an explicit relation between interquark potential and inflation. We show explicitly that the interquark potential in the confining phase depends on the Hubble parameter and on the tension of the braneworld. The Hubble parameter here is taken as a constant. This is a signature of an inflationary phase in the braneworld. Furthermore, in this inflationary scenario the Hubble parameter can be seen as an order parameter. When the Hubble parameter is zero the interquark potential is Coulomb-like, indicating a deconfining phase. As the Hubble parameter is turned on, the confining phase is turned on.

In the present work we shall regard a delta function source for a single inflating 3-brane at $r=0$ with tension σ as the place where there are N coincident ‘deformed $D3$ -branes’ generating the same background [24]. We identify the $D3$ -brane tension $T_{D3} = \sigma$. At the boundary of this spacetime generated by such deformed $D3$ -branes, we have a four-dimensional Euclidean AdS_4 geometry where the Wilson loop is evaluated and so does the heavy quark-anti-quark potential associated with the dual gauge theory.

This paper is organised as follows: in section 2, we describe the brane cosmology scenario we are going to work with; in section 3, we consider an inflating braneworld embedded in a static AdS_5 and then obtain the interquark distance integral and the action integral; in section 4, we set the large N limit (or large D -brane tension) and describe the cases in which the interquark distance and the action should be evaluated; in section 5, we obtain the interquark potential in the cases delimited in the previous section. In particular, we explicit show the Hubble parameter dependence of the interquark potential in the confining phase. We close this paper with some conclusions and perspectives.

2 Brane cosmology background

The metric due to a stack of a large number N of $D3$ -branes provides a nice gravity dual for a gauge field theory developing both confining and non-confining regimes. This background is a ten-dimensional spacetime with metric

$$ds^2 = \left(1 + \frac{R^4}{U^4}\right)^{-1/2} (-dt^2 + d\vec{x}^2) + \left(1 + \frac{R^4}{U^4}\right)^{1/2} (dU^2 + U^2 d\Omega_5^2). \quad (2.1)$$

Sufficiently close to the brane, i.e., $U \ll R$ the space has the $AdS_5 \times S^5$ topology. As usual R is the AdS_5 radius (the same as the S^5 radius) and $d\Omega_5$ is the S^5 metric. Disregarding the S^5 part, we are left with an AdS_5 spacetime with metric

$$ds^2 = \alpha' \left[\frac{U^2}{R^2} (-dt^2 + d\vec{x}^2) + \frac{R^2}{U^2} dU^2 \right], \quad (2.2)$$

rescaled in terms of string units, whose boundary at $U \rightarrow \infty$ is a four-dimensional Minkowski space. One may euclideanize the boundary of the spacetime in order to calculate a Wilson loop in this boundary. The Wilson loop area is related to the string world-sheet area. The five-dimensional spacetime in this case is considered as an Euclidean AdS_5 space [31, 32].

Similar spaces are obtained in braneworld scenarios with inflation [18, 19, 24] or without inflation [33] — see [7] for developments on gauge/gravity duality in the Randall-Sundrum scenario [33]. Braneworld scenarios are found by considering a delta function source in a five-dimensional gravity theory given by the action

$$S = -\frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} (R + \Lambda_{\text{bulk}}) + \sigma \int d^5x \sqrt{-g} \delta(r), \quad (2.3)$$

where it is assumed that $\kappa_4^2 = 8\pi G_N \simeq \kappa_5^4 \sigma / 6 = 2$, with G_N being Newton's constant and σ is going to be associated with the brane tension. The five-dimensional cosmological constant is defined as $\rho_{\text{bulk}} \equiv \Lambda_{\text{bulk}} = -\sigma^2 \kappa_5^2 / 6$, which satisfies the Randall-Sundrum fine tuning [33]. For a brane cosmology with an inflating 3-brane [18, 19, 24] the spacetime general metric is of the form

$$ds^2 = -n^2(t, r) dt^2 + a^2(t, r) \gamma_{ij} dx^i dx^j + b^2(t, r) dr^2, \quad (2.4)$$

where γ^{ij} is a maximally symmetric 3-dimensional metric with spatial curvature $k = -1, 0, 1$. The cosmological solution for a 3-brane inflating along t, x^i , but static in r (i.e., along the bulk) [18, 19] reads

$$\begin{aligned} a(t, r) &= a_0(t) \left[\frac{1}{2} \left(1 + \frac{\kappa_5^2 \rho_b^2}{6\rho_{\text{bulk}}} \right) + \frac{1}{2} \left(1 - \frac{\kappa_5^2 \rho_b^2}{6\rho_{\text{bulk}}} \right) \cosh(\mu r) - \frac{\kappa_5 \rho_b}{\sqrt{-6\rho_{\text{bulk}}}} \sinh(\mu|r|) \right]^{1/2}, \\ n(t, r) &= \frac{\dot{a}(t, r)}{\dot{a}_0(t)}, \\ b(t, r) &= 1, \end{aligned} \quad (2.5)$$

where $\mu = (1/3)(\kappa_5^4 \sigma^2)^{1/2}$, a_0 is the scale factor on the brane and ρ_b is the brane energy density. The radiation ‘ \mathcal{C} -term’ and the curvature ‘ k -term’ have been disregarded. The dynamics on the 3-brane [24] is governed by the induced Friedmann equation on the brane

$$H^2 = \frac{2}{3} \rho \left(1 + \frac{\rho}{2\sigma} \right), \quad (2.6)$$

where $H = \dot{a}_0/a_0$, with $a_0 = a(t, r=0)$ being the scale factor on the brane worldvolume with metric

$$ds_4^2 = -dt^2 + a_0^2(t) (dx^2 + dy^2 + dz^2), \quad (2.7)$$

where we have assumed that $\rho_b = \rho + \sigma$, with σ being the brane tension and ρ being a cosmological energy in the brane.

For $r \gg R \sim 1/\mu$, with R being a characteristic radius of the bulk, one may write $a(t, r) = U(r)a_0(t) \sim e^{r/R}a_0(t)$. If $U/R \ll 1$ then $dU = (U/R)dr \sim 0$, and the metric (2.4) can be reduced to a metric similar to that of a space due to a large number N of $D3$ -branes (2.2), except for the fact that its boundary at $U \rightarrow \infty$ is *not* a flat spacetime. We anticipate that the original metric (2.4) with the solutions (2.5) may be written in the general form²

$$ds^2 = \alpha' \left[\frac{U^2}{R^2} (-dt^2 + a_0(t)^2 d\vec{x}^2) + \frac{R^2 U^2}{(U^2 - C_1)(U^2 - C_2)} dU^2 \right], \quad (2.8)$$

with $C_{1,2}$ being proportional to the brane properties, for instance its tension σ and its geometry — see below for further details. Particularly in the inflationary case, the space has a positive cosmological constant, i.e., a four-dimensional de Sitter space, with Lorentzian signature. In the boundary, the Wilson loop is calculated in the Euclideanized space, that means an ‘Euclidean AdS_4 ’ space. The five-dimensional spacetime in this case is considered as an ‘Euclidean AdS_4 - AdS_5 ’ space [31, 32].

3 The gauge/gravity duality for an inflating brane background

In this section we analyse the case in which H , the Hubble parameter for the braneworld is a constant. This is a signature of an inflationary phase in the braneworld.

For future reference, we begin by rewriting the metric (2.4) in the general form

$$ds^2 = n^2(t, r) e dt^2 + a^2(t, r) \gamma_{ij} dx^i dx^j + b^2(t, r) dr^2, \quad (3.1)$$

where $e = -1$ means Lorentzian signature and $e = +1$ (i.e. $t \mapsto i\tau$) means Euclidean signature. In this case we may rewrite the scale factor in (2.5) as

$$a(t, r) = U(r)a_0(t), \quad (3.2)$$

where the “warp factor” reads

$$U(r) = \left(e\gamma + (1 - e\gamma) \cosh(\mu r) - \sqrt{1 - 2e\gamma} \sinh(\mu|r|) \right)^{1/2}, \quad (3.3)$$

with

$$\gamma = \frac{3H^2}{2\sigma} \quad \text{when } e = -1 \quad \text{or} \quad (3.4)$$

$$\gamma = \frac{3H_E^2}{2\sigma} \quad \text{when } e = +1, \quad (3.5)$$

with $H_E = \frac{\dot{a}_0(\tau)}{a_0(\tau)}$, where the subindex E stands for Euclidean, and σ being the 3-brane tension. Observe that $H \rightarrow iH_E$ or $\gamma \rightarrow -\gamma$ when $e = -1 \mapsto e = +1$.

²There is a suitable rescaling of U and R so that this metric can be compared with (2.2). For further details see ahead the transformations (4.6).

In the Euclidean case ($e = +1$), we mostly consider this geometry as an asymptotically ($\mu r \rightarrow \infty$) Euclidean AdS_4 - AdS_5 space. We also consider the Euclidean Hubble parameter $H_E^2 = \frac{2}{3}\gamma\sigma = \text{const.}$. This implies that $a_0(\tau) = \exp(\pm\sqrt{\bar{\Lambda}}\tau)$, where $\bar{\Lambda} = \frac{2}{3}\gamma\sigma$ is defined as the cosmological constant on the Euclidean brane. As we observe later, in order to ensure leading string contributions, the brane tension should be very large. Thus we are going to analyse the case when $\sigma \rightarrow \infty$. In this case, we may have three possible regimes for γ depending on H_E , i.e.,

1. $\gamma \rightarrow 0$, when H_E is held fixed,
2. γ is finite, when H_E^2 goes to infinity as fast as σ goes to infinity, and
3. $\gamma \rightarrow \infty$, when H_E^2 goes to infinity faster than σ goes to infinity.

Later we will see how γ is related to a confining phase for the interquark potential. In order to write the interquark distance and the action in terms of the string world-sheet we follow [5] and define

$$f(\tau, r) = \sqrt{g_{\tau\tau} g_{xx}} = U(r)^2 a_0(\tau) \equiv f(r) a_0(\tau), \tag{3.6}$$

$$g(\tau, r) = \sqrt{g_{\tau\tau} g_{rr}} = U(r) \equiv g(r), \tag{3.7}$$

where the subindices τ , x and r in the metric $g_{\mu\nu}$ stand for the (Euclidean) time coordinate, the coordinate along the brane and the coordinate tranverse to the brane, respectively. Now, the dependence of the interquark distance L with respect to $a_0(\tau)$ is

$$L(\tau) = 2 \int_{r_0}^{r_1} dr \frac{g(\tau, r)}{f(\tau, r)} \frac{f(\tau, r_0)}{\sqrt{f(\tau, r)^2 - f(\tau, r_0)^2}} = L_0 \frac{1}{a_0(\tau)}, \tag{3.8}$$

where L_0 is the static length given by

$$\begin{aligned} L_0 &= 2 \int_{r_0}^{r_1} dr \frac{g(r)}{f(r)} \frac{f(r_0)}{\sqrt{f(r)^2 - f(r_0)^2}}, \\ &= 2 \int_{r_0}^{r_1} dr \frac{1}{U(r)} \frac{U(r_0)^2}{\sqrt{U(r)^4 - U(r_0)^4}}. \end{aligned} \tag{3.9}$$

The limits r_0 fixed and $r_1 \rightarrow \infty$ stand for the place where $U(r)$ develops a minimum and the boundary of the five-dimensional spacetime, respectively. As we see shortly, despite of the dependence of the length (3.8) on the scale $a_0(\tau) = \exp(\pm\sqrt{\bar{\Lambda}}\tau)$, the area of the string world-sheet is invariant under such a scaling. This clearly generates a conflict. Roughly speaking, this happens because the area in the expectation value of the Wilson loop

$$\langle W(\mathcal{C}) \rangle \sim e^{-\text{const.} L_0 \int_0^T d\tau / a_0(\tau)}, \tag{3.10}$$

diverges exponentially, whereas the string world-sheet area diverges linearly as $T \rightarrow \infty$. Thus, in order to circumvent this problem we keep the calculations of the Wilson loop in a flat spacetime at the boundary $r = r_1$, by assuming that the Wilson loop is being

calculated in a “comoving frame”. This is the frame with static length $L_0 = a_0(\tau)L(\tau)$ given in (3.9). Therefore one ensures an area law for

$$\langle W(\mathcal{C}) \rangle \sim e^{-\text{const.}L_0 T}, \tag{3.11}$$

that gives a confining phase. Alternatively, one could also consider calculations by imposing a stringent bound on $\bar{\Lambda}$, i.e. $\sqrt{\bar{\Lambda}}T \ll 1$, such that we could use $L(\tau_0) \simeq L_0$. However, this is precisely the limit of a flat four-dimensional space on the 3-brane, where $\gamma \simeq 0$, or simply the AdS_5 limit which is a conformal space and is not a good gravity dual for a gauge field theory developing a confining regime.

As we have anticipated, the string world-sheet action (i.e. the Nambu-Goto action) is not dependent on the scale $a_0(\tau)$. After few calculations, one can show that it is written as

$$\begin{aligned} S &= \frac{1}{2\pi\alpha'} \int_{r_0}^{r_1} \int_0^T dr d\tau \frac{g(r, \tau) f(r, \tau)}{\sqrt{f(r, \tau)^2 - f(r_0, \tau)^2}}, \\ &= \frac{T}{2\pi\alpha'} \int_{r_0}^{r_1} dr \frac{U(r)^3}{\sqrt{U(r)^4 - U(r_0)^4}}, \end{aligned} \tag{3.12}$$

Observe that the dependence of the integrand with respect to τ has disappeared. The area enclosed by some square contour \mathcal{C} in (3.11) can be scaled up by $a_0(\tau)$, whereas the world-sheet area in (3.12) cannot be scaled up by $a_0(\tau)$. A similar effect has been discussed in [11] in the AdS/CFT context, where the conformal invariance guarantees to scale up the area of the contour without changing the world-sheet area in the bulk.

Because the functions $f(r) \equiv f(r, \tau)/a_0(\tau)$ and $g(r) \equiv g(r, \tau)$ made out of the metric components diverge as $r \rightarrow \infty$, the action (3.12) also diverges with $\int^\infty g(r) dr$. We may relate this fact to the quark mass, that we should subtract from the action (3.12) [1, 5]. So we define the regularized quark mass as

$$m_q = \frac{1}{2\pi\alpha'} \int_0^{r_1} g(r) dr = \frac{1}{2\pi\alpha'} \left[\int_0^{r_0} g(r) dr + \int_{r_0}^{r_1} g(r) dr \right], \tag{3.13}$$

such that the regularized action $S^{\text{reg}} = S - \frac{\sqrt{e}}{2\pi\alpha'} \int m_q d\tau$ is written as

$$S^{\text{reg}} = \frac{\sqrt{e}T}{2\pi\alpha'} \left[\int_{r_0}^{r_1} dr \left(\frac{g(r)f(r)}{\sqrt{f(r)^2 - f(r_0)^2}} - g(r) \right) - \int_0^{r_0} g(r) dr \right], \tag{3.14}$$

where we set $r_1 \rightarrow \infty$. Recall that from equation (3.7) the function g is τ independent, so does the quark mass m_q .

4 Large N limit (or large D -brane tension)

Since we are interested in the weak version of the gauge/gravity duality [34, 35], we take into account only the leading string contributions, where gravity description is valid. In this limit we have that the string coupling $g_s \rightarrow 0$ and the number of D -branes $N \rightarrow \infty$, while $g_s N$ is held constant and sufficiently large. As a consequence the $D3$ -brane tension

$T_{D3} = (2\pi)^{-3} l_s^{-4} g_s^{-1}$ should be very large. Notice that we try to maintain almost all the ingredients of the AdS/CFT conjecture, such that at least the holographic principle of gauge theory (on the boundary) corresponding to gravity (on the bulk) is guaranteed.

Hereafter we assume that $T_{D3} = \sigma$. Since $\mu \sim \sqrt{k^4 \sigma^2}$, we have that μ is sufficiently large. This simplifies our computations by considering μr also very large for $r > 0$. Thus, for³ $r > 0$, we may rewrite the warp factor as

$$U(r) = (e\gamma + \xi^2 e^{\mu r} + \chi^2 e^{-\mu r})^{\frac{1}{2}}, \tag{4.1}$$

where

$$\xi^2 = \frac{1}{2}(1 - e\gamma - \sqrt{1 - 2e\gamma}), \quad \chi^2 = \frac{1}{2}(1 - e\gamma + \sqrt{1 - 2e\gamma}). \tag{4.2}$$

Now, the derivative of the warp factor is given by

$$U' = \frac{1}{R_0 U} \sqrt{(U^2 - e\gamma)^2 - 4\xi^2 \chi^2}, \tag{4.3}$$

with a bulk characteristic radius defined as

$$R_0 = \frac{2}{\mu}. \tag{4.4}$$

Thus, we have the helpful relation

$$dr = \frac{1}{U'} dU = \frac{R_0 U}{\sqrt{(U^2 - e\gamma)^2 - 4\xi^2 \chi^2}} dU. \tag{4.5}$$

It is useful to rescale U to have dimension of energy by rearranging the dimensions of the quantities in the metric through the following transformations

$$\begin{aligned} U &\rightarrow \frac{\alpha'}{R_0} U, \\ R_0^2 &= \alpha' R^2, \end{aligned} \tag{4.6}$$

where α' is related to the inverse of the string tension. We now consider the following changing of coordinate

$$y = \frac{U}{U_0}, \quad \text{with } U_0 \equiv U(r_0), \tag{4.7}$$

with r_0 being where $U(r)$ assumes its minimum value.

In terms of the above transformations, i.e., (4.5), (4.6) and (4.7) the metric reads⁴

$$ds^2 = \alpha' \left[\frac{U_0^2}{R^2} y^2 (e dt^2 + a_0(t)^2 d\vec{x}^2) + \frac{R^2 y^2}{(y^2 - C_+)(y^2 - C_-)} dy^2 \right], \tag{4.8}$$

with

$$C_{\pm} = (e\gamma \pm 2|\xi\chi|) \frac{R^2}{\alpha' U_0^2}, \tag{4.9}$$

³There is another branch [24], $r \leq 0$, that we disregard in the present investigations.

⁴According to transformations (4.6), the dimensions of the quantities in this metric are the following: $[\alpha'] = (\text{length})^2$, $[U] = (\text{length})^{-1}$, $[R] = 1$, $[\tau] = [\vec{x}] = \text{length}$.

where ξ and χ is given in (4.2). Notice that as $\gamma \rightarrow 0$ equations (4.2) and (4.9) tell us that the ‘deforming constants’ $C_{\pm} \rightarrow 0$. Furthermore equations (3.4) and (3.5) say that in this regime $a_0(t)$ is just a constant we set equal to 1. As a consequence the AdS space (2.2) is recovered from (4.8). As we see later, considering γ small we find the interquark Cornell potential perturbatively in C_{\pm} . The leading term is the same found in the original AdS/CFT correspondence. The limit $y \rightarrow \infty$ recovers the boundary of the five-dimensional space as the four-dimensional Euclidean AdS_4 space (for $e = +1$) that we may identify as the induced metric on the 3-brane.

We now proceed to evaluate the interquark distance and the action.

By using (3.9) and the transformations (4.5), (4.6) and (4.7), we find that the static interquark distance is

$$L_0 = 2 \frac{R^2}{U_0} \int_1^{U_1/U_0} dy \frac{1}{\sqrt{y^4 - 1}} \frac{1}{\sqrt{(y^2 - C_+)(y^2 - C_-)}}. \quad (4.10)$$

By using (3.12) and the transformations (4.5), (4.6) and (4.7), we find that the action is

$$S = \frac{\sqrt{e} T U_0}{2\pi} \int_1^{U_1/U_0} dy \frac{y^4}{\sqrt{y^4 - 1}} \frac{1}{\sqrt{(y^2 - C_+)(y^2 - C_-)}}. \quad (4.11)$$

As a first case, we can easily see that that the limits $C_{\pm} \rightarrow 0$ into the metric (4.8) recover the AdS_5 metric and the limit $y \rightarrow \infty$ recovers the AdS_5 boundary.

Since

$$\xi^2 \chi^2 = \frac{1}{4} ((e\gamma)^2 + 1 - 2e\gamma - |1 - 2e\gamma|), \quad (4.12)$$

which appears in the definition of C_{\pm} in (4.9), we may consider the following two cases:

1. If $e\gamma \leq \frac{1}{2}$, then

$$\xi^2 \chi^2 = \frac{(e\gamma)^2}{4}, \quad (4.13)$$

and

$$C_{\pm} = (e \pm 1) \gamma \frac{R^2}{\alpha' U_0^2}. \quad (4.14)$$

2. If $e\gamma > \frac{1}{2}$, then

$$\xi^2 \chi^2 = \frac{1}{4} (\gamma^2 + 2(1 - 2e\gamma)), \quad (4.15)$$

and

$$C_{\pm} = \left(e\gamma \pm \sqrt{(e\gamma - 2)^2 - 2} \right) \frac{R^2}{\alpha' U_0^2} \quad (4.16)$$

5 Results

In this section we analyse the two possible regimes for $e\gamma$ as stated in the end of the last section. From now on we mainly work in the Euclidean case, $e = +1$. In this case we show how a deconfining phase and a confining phase appear. We also evaluate the Cornell

potential. As a last result we comment on the Lorentzian case, $e = -1$, and conjecture on interesting possible phenomenology for meson pair production.

Following the common practice initiated in [1], we evaluate both integrals for the action and interquark distance in the Euclidean space. Thus, we have for

1. $\gamma \leq \frac{1}{2}$ that

$$C_+ = 2\gamma \frac{R^2}{\alpha' U_0^2}, \tag{5.1}$$

$$C_- = 0, \tag{5.2}$$

and for

2. $\gamma > \frac{1}{2}$ that

$$C_{\pm} = \left(\gamma \pm \sqrt{(\gamma - 2)^2 - 2} \right) \frac{R^2}{\alpha' U_0^2} = \left(1 \pm \sqrt{\left(1 - \frac{2}{\gamma}\right)^2 - \frac{2}{\gamma^2}} \right) \gamma \frac{R^2}{\alpha' U_0^2}. \tag{5.3}$$

Here we may consider $\gamma \rightarrow \infty$, so that

$$C_{\pm} = (1 \pm 1) \gamma \frac{R^2}{\alpha' U_0^2}, \tag{5.4}$$

which simplifies to

$$C_+ = 2\gamma \frac{R^2}{\alpha' U_0^2} \tag{5.5}$$

$$C_- = 0. \tag{5.6}$$

In summary, for any of the two cases above, the integrals we have to study simply become

$$L_0 = 2 \frac{R^2}{U_0} \int_1^{U_1/U_0} dy \frac{1}{y \sqrt{y^4 - 1}} \frac{1}{\sqrt{y^2 - C_+}}, \tag{5.7}$$

$$S = \frac{T U_0}{2\pi} \int_1^{U_1/U_0} dy \frac{y^3}{\sqrt{y^4 - 1}} \frac{1}{\sqrt{y^2 - C_+}}. \tag{5.8}$$

In the sequel, we shall evaluate the interquark potential using the above integrals. First we obtain a Coulomb-like phase (deconfining phase). Next we obtain a confining phase.

5.1 A Coulomb-like phase

From now on, we consider

$$\gamma \leq 1/2, \quad C_+ = 2\gamma \frac{R^2}{\alpha' U_0^2}, \tag{5.9}$$

so that the interquark distance is now

$$L_0 = 2 \frac{R^2}{U_0} \int_1^{U_1/U_0} dy \frac{1}{y \sqrt{y^4 - 1}} \frac{1}{\sqrt{y^2 - C_+}} \stackrel{U_1 \rightarrow \infty}{=} 2 \frac{R^2}{U_0} I_1(C_+). \tag{5.10}$$

The regularized action is now

$$S^{\text{reg}} = \frac{T U_0}{2\pi} \left[\int_1^{U_1/U_0} dy \left(\frac{y^3}{\sqrt{y^4-1}} \frac{1}{\sqrt{y^2-C_+}} - 1 \right) - 1 \right] \stackrel{U_1 \rightarrow \infty}{=} \frac{T U_0}{2\pi} I_2(C_+), \quad (5.11)$$

where $I_{1,2}(C_+)$ being definite integrals for $U_1 \rightarrow \infty$.

Now, for $C_+ = 0$ (i.e., $\gamma = 0$), we solve (5.10) for U_0 , and substitute it into (5.11) in order to find

$$S^{\text{reg}} \equiv TE(L_0) = \frac{T}{\pi} I_1(0)I_2(0) \frac{R^2}{L_0} \longrightarrow E(L_0) = -\frac{2\pi^2}{\Gamma(1/4)^4} \frac{R^2}{L_0}. \quad (5.12)$$

Thus, the regularized action (5.11) gives the interquark potential in a Coulomb-like phase. This case gives precisely the Maldacena's results for AdS/CFT correspondence [1], with $R_0^4 \equiv R^4 \alpha'^2 = 4\pi g_s N \alpha'^2 = 4\pi g_{\text{YM}}^2 N \alpha'^2$, with $g_{\text{YM}}^2 = g_s$. Recall that the Maldacena limit is defined as R large and fixed, when $N \rightarrow \infty$, while $\alpha' \rightarrow 0$. This is consistent with $R_0 \rightarrow 0$, since either $R_0 = 2/\mu$ and $\mu \rightarrow \infty$ or $R_0^2 = \alpha' R^2$ and $\alpha' \rightarrow 0$ with R large and fixed.

5.2 Opening up a confining phase

Let us now consider that $C_+ = 1$, i.e., $U_0^2 = 2\gamma \frac{R^2}{\alpha'}$. This case reveals a new regime in the calculations above. Firstly we note that the integral (5.7) becomes now

$$L_0 = \frac{\alpha' U_0}{\gamma} \int_1^{U_1/U_0} dy \frac{1}{y\sqrt{y^4-1}} \frac{1}{\sqrt{y^2-1}} \stackrel{U_1 \rightarrow \infty}{=} \frac{\alpha' U_0}{\gamma} I_1(1), \quad (5.13)$$

(note that the prefactor of L_0 has changed since $2R_0^2/U_0 \rightarrow \frac{\alpha' U_0}{\gamma}$), and the regularized action is now

$$S^{\text{reg}} = \frac{T U_0}{2\pi} \left[\int_1^{U_1/U_0} dy \left(\frac{y^3}{\sqrt{y^4-1}} \frac{1}{\sqrt{y^2-1}} - 1 \right) - 1 \right] \stackrel{U_1 \rightarrow \infty}{=} \frac{T U_0}{2\pi} I_2(1), \quad (5.14)$$

where $I_{1,2}(C_+ = 1)$ being definite integrals for $U_1 \rightarrow \infty$. Just as in the previous calculation, we can easily show that the regularized action (5.14) written in terms of the length L_0 (5.13) is

$$S^{\text{reg}} = T \frac{I_2(1)}{I_1(1)} \frac{\gamma}{2\pi\alpha'} L_0. \quad (5.15)$$

The integrals $I_{1,2}(1)$ are regularized only in the UV-regime. There is a pole at $y = 1$ that should also be regularized. This can easily be done by considering an infrared cut-off ε that can be set equal to '1' in the end of the computations (see appendix). We anticipate that this procedure will reveal that the rate $\frac{I_2(1)}{I_1(1)} \rightarrow 1$ as $\varepsilon \rightarrow 1$. Thus, we end up with the confining regime

$$S^{\text{reg}} \equiv E(L_0)T = T \frac{\gamma}{2\pi\alpha'} L_0 \longrightarrow E(L_0) = \frac{\gamma}{2\pi\alpha'} L_0, \quad (5.16)$$

where $\gamma/2\pi\alpha'$ plays the role of a "QCD string" tension, that depends on the $D3$ -brane tension and the Hubble parameter. We note that this result can also be written as

$$E(L_0) = \frac{\gamma a_0(\tau)}{2\pi\alpha'} L(\tau). \quad (5.17)$$

where we have used $L_0 = a(\tau)L(\tau)$. This generalizes the confining static term $E(L_0) = f(r_0)L_0$ obtained in the literature. In the present case, one still have a static confining term that depends on the cosmological constant $\bar{\Lambda} = H_E^2 \sim \gamma\sigma$ on the $D3$ -brane where the field theory lives.

5.3 Cornell potential

Let us now consider the equation (5.7) and (5.8) to evaluate the Cornell potential. Here we expand these integrals in powers of C_+ with

$$\gamma \leq 1/2, \quad C_+ = 2\gamma \frac{R^2}{\alpha' U_0^2}. \quad (5.18)$$

For the sake of simplicity we are just considering the leading term in the interquark distance L_0 i.e.

$$L_0 = 2 \frac{R^2}{U_0} \int_1^{U_1/U_0} dy \frac{1}{y^2 \sqrt{y^4 - 1}} + O(C_+) \stackrel{U_1 \rightarrow \infty}{=} 2 \frac{R^2}{U_0} I_1(0) + O(C_+), \quad (5.19)$$

and regularized action power expanded up to the second order in C_+

$$S^{\text{reg}} = \frac{T U_0}{2\pi} \left[\int_1^{U_1/U_0} dy \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) - 1 + \frac{C_+}{2} \int_1^{U_1/U_0} dy \frac{1}{\sqrt{y^4 - 1}} + \frac{3C_+^2}{8} \int_1^{U_1/U_0} dy \frac{1}{y^2 \sqrt{y^4 - 1}} \right] + O(C_+^3). \quad (5.20)$$

Thus,

$$E(L_0) = \frac{I_1 I_2 R^2}{2\pi L_0} + \frac{I_3 \gamma L_0}{2I_1 2\pi\alpha'} + \frac{3}{16} \frac{1}{I_1^2} \frac{\gamma^2 L_0^3}{2\pi\alpha'^2 R^2}, \quad (5.21)$$

where

$$\begin{aligned} I_1 &= \int_1^{U_1/U_0} dy \frac{1}{y^2 \sqrt{y^4 - 1}} = 1/2 \frac{\Gamma(3/4)^2 \sqrt{2}}{\sqrt{\pi}} \stackrel{U_1 \rightarrow \infty}{=} 0.599070117, \\ I_2 &= \int_1^{U_1/U_0} dy \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) - 1 \stackrel{U_1 \rightarrow \infty}{=} -1/2 \frac{\Gamma(3/4)^2 \sqrt{2}}{\sqrt{\pi}} = -I_1, \\ I_3 &= \int_1^{U_1/U_0} dy \frac{1}{\sqrt{y^4 - 1}} = 1/4 \frac{\pi^{3/2} \sqrt{2}}{\Gamma(3/4)^2} \stackrel{U_1 \rightarrow \infty}{=} 1.311028777. \end{aligned}$$

Thus,

$$E(L_0) = \frac{1}{2\pi\alpha'} \left[-0.7177700096 \frac{\alpha' R^2}{L_0} + 1.094219807 L_0 \gamma + 0.5224514746 \frac{L_0^3 \gamma^2}{\alpha' R^2} \right] \quad (5.22)$$

This is the well-known [36] Cornell potential $E(r) = -a/r + br$, where a, b are non-negative constants and r is the interquark distance. This potential is commonly used to describe the physics of heavy quarks. One can successfully obtain the whole mass spectrum of the quark anti-quark pair in the quarkonium system using this potential. In the confining term in (5.22), there appears γ that depends on the Euclidean Hubble parameter H_E and the brane tension σ . Thus ‘QCD string’ tension is $\sim \gamma/\alpha'$.

5.4 A Lorentzian signature limit

There is another interesting limit. In the limit of $C_{\pm} \rightarrow \infty$ ($\gamma \rightarrow \infty$) the Nambu-Goto action is well defined only in a ‘Lorentzian space’, i.e. $e = -1$, where $C_+ \rightarrow C_-$. Thus, the leading term of the properly regularized action (5.20) in the limit $C_- \rightarrow \infty$ reads

$$S^{\text{reg}} = \frac{\sqrt{e}}{\sqrt{-C_-}} \frac{T U_0}{2\pi} \int_1^{U_1/U_0} dy \left(\frac{y^3}{\sqrt{y^4 - 1}} - y \right) = \frac{\sqrt{e}}{2\sqrt{-C_-}} \frac{TU_0}{2\pi} = \frac{\sqrt{\alpha'}}{2\sqrt{2}\gamma} \frac{iTU_0^2}{2\pi R}, \quad (5.23)$$

where we have used the previous result $C_{\pm} = (e \pm 1) \frac{\gamma R^2}{\alpha' U_0^2}$ and the fact that $\sqrt{e} = i$ for the Lorentzian signature. Thus the mass of a meson pair is obtained by using the Wilson loop evaluated in the Lorentzian signature

$$\langle W(\mathcal{C}) \rangle \sim e^{iTE(L)} = e^{S^{\text{reg}}}, \quad (5.24)$$

such that

$$E(L) = \frac{\sqrt{\alpha'}}{\sqrt{2|\gamma|}} \frac{U_0^2}{4\pi R}. \quad (5.25)$$

Recall that U_0 assumes its minimum value at r_0 in the AdS_5 space. We can see that in the limit of $C_{\pm} \rightarrow \infty$ in (5.7) U_0 does not depend on L_0 . Thus, we can ensure that the potential in (5.25) does not depend on the interquark distance L_0 and is therefore constant. This regime is usually assumed to be favorable to meson pairs creation from the vacuum [9, 37]. This is a regime where the Wilson loop develops a ‘perimeter law’, in contrast to the area law in the confining regime.

6 Conclusions and perspectives

In this paper we have considered the five-dimensional space, the brane cosmology background, as a gravity dual of a confining gauge field theory living on its boundary. This space develops the precise phenomenon of deforming a non-confining conformal AdS geometry into a confining geometry described by an asymptotically Euclidean $AdS_4 - AdS_5$ space. The main result shows that the cosmological constant on the brane is directly related to the confining term of the interquark potential.

The well-known Cornell potential found in earlier proposals for AdS/QCD correspondence [8] was also here obtained. It is the kind of potential we can use to describe heavy quarks. One can successfully obtain the whole mass spectrum of the quark anti-quark pair in the quarkonium system using such a potential. Thus, in our setup it is possible to estimate the suitable regime for the cosmological constant favoring the meson pair production during the evolution of the Universe as $(\gamma/\alpha')L_0 \rightarrow \text{const.}$, with L_0 being the typical meson size in this regime. Several other investigations can also be done such as relating the number of e-folds with the confining phase.

We have also obtained a regime where the Wilson loops shows up a ‘perimeter law’. As pointed out in [9, 37], it is possible to consider meson pair creation in this regime. Furthermore, as one can see in (5.25), this regime also depends on the Hubble parameter.

In summary, in our setup we show an expected phenomenology described in the dual field theory for the Universe in the inflationary brane. As one knows, in the inflationary regime, the Universe is cooling. Therefore there is a phase transition in QCD - from a phase of nearly free quarks and gluons to a hadronic phase. Thus, it is tempting to imagine that as the Universe expands quarks may confine, in some delicate manner, because of the accelerated expansion of the Universe due to a cosmological constant.

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A Evaluating the integrals $I_{1,2}(1)$

Let us use the infrared cut-off ε in order to properly evaluate the integrals $I_{1,2}(1)$. The first integral is

$$I_1(1) = \int_{\varepsilon}^{\infty} dy \frac{1}{y\sqrt{y^4-1}} \frac{1}{\sqrt{y^2-1}} = 1/4\sqrt{2}\operatorname{arctanh}\left(1/2\frac{\sqrt{2}\varepsilon}{\sqrt{\varepsilon^2+1}} + 1/2\frac{\sqrt{2}}{\sqrt{\varepsilon^2+1}}\right) - 1/4\sqrt{2}\operatorname{arctanh}\left(1/2\frac{\sqrt{2}\varepsilon}{\sqrt{\varepsilon^2+1}} - 1/2\frac{\sqrt{2}}{\sqrt{\varepsilon^2+1}}\right) - \operatorname{arctanh}\left(\frac{1}{\sqrt{\varepsilon^2+1}}\right), \quad (\text{A.1})$$

and

$$I_2(1) = \int_{\varepsilon}^{\infty} dy \left(\frac{y^3}{\sqrt{y^4-1}} \frac{1}{\sqrt{y^2-1}} - 1\right) - 1 = -\frac{\varepsilon^2}{\sqrt{\varepsilon^2+1}} - \frac{1}{\sqrt{\varepsilon^2+1}} + 1/4\sqrt{2}\operatorname{arctanh}\left(1/2\frac{\sqrt{2}\varepsilon}{\sqrt{\varepsilon^2+1}} + 1/2\frac{\sqrt{2}}{\sqrt{\varepsilon^2+1}}\right) - 1/4\sqrt{2}\operatorname{arctanh}\left(1/2\frac{\sqrt{2}\varepsilon}{\sqrt{\varepsilon^2+1}} - 1/2\frac{\sqrt{2}}{\sqrt{\varepsilon^2+1}}\right) + \varepsilon - 1. \quad (\text{A.2})$$

In the limit $\varepsilon \rightarrow 1$ both integral goes to infinity with the same dominant term:

$$\operatorname{arctanh}\left(1/2\frac{\sqrt{2}\varepsilon}{\sqrt{\varepsilon^2+1}} + 1/2\frac{\sqrt{2}}{\sqrt{\varepsilon^2+1}}\right) \rightarrow \infty. \quad (\text{A.3})$$

Thus, recovering the original limit of the integral one finds $\frac{I_1(1)}{I_2(1)} \rightarrow 1$.

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